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ScienceDirect
International Journal of Heat and Fluid Flow 27 (2006) 831–837
International Journal of HEAT AND FLUID FLOW
www.elsevier.com/locate/ijhff

Behaviour of turbulence models near a turbulent/non-turbulent interface revisited

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Received 30 September 2005; received in revised form 23 January 2006; accepted 4 March 2006
Available online 15 June 2006

Abstract

The behaviour of turbulence models near a turbulent/non-turbulent interface is investigated. The analysis holds as well for two-equation as for Reynolds stress turbulence models using Daly and Harlow diffusion model.

The behaviour near the interface is shown not to be a power law, as usually considered, but a more complex parametric solution. Why previous works seemed to numerically confirm the power law solution is explained.

Constraints for turbulence modelling, i.e., for ensuring that models have a good behaviour near a turbulent/non-turbulent interface so that the solution is not sensitive to small turbulence levels imposed in the irrotational flow, are drawn.

Keywords: Eddy viscosity models; Model constraints; Reynolds stress models; Turbulence modelling

1. Introduction

A well-known drawback of some turbulence models such as the Wilcox (1988) $k-\omega$ model, the Smith (1995) $k-L$ model or the Baldwin and Barth (1991) one-equation model is that the solution is sensitive to the small level of transported turbulent quantities imposed outside of the turbulent regions. Indeed, for low turbulence levels outside of the boundary or free shear layers, turbulence should propagate from the turbulent regions towards the non-turbulent regions, a condition which is violated by these models. The solution thus depends upon the small level of eddy viscosity outside of the turbulent region, i.e., upon the external turbulence length scale. Such an unphysical behaviour cannot be accepted as the numerical solutions are thus unreliable. It must be reminded that this problem is different from the sensitivity to free-stream turbulence, in which significant levels of turbulence outside of the boundary or free shear layers are considered.

The real physical behaviour near a turbulent/non-turbulent interface is rather complex, with an interface which is highly corrugated and induces fluctuations in the irrotational, non-turbulent field. Describing it is out of the capabilities of standard turbulence models which all more or less rely upon turbulence equilibrium assumptions. Therefore, all what is required is that the turbulence model correctly propagates information from the turbulent towards the non-turbulent region to avoid undesirable free-stream sensitivity.

2. Standard analysis

For the sake of simplicity, only incompressible flows will be addressed.

The behaviour near a turbulent/non-turbulent interface was first addressed by Saffman (1970) and deeper investigated by Cazalbou et al. (1994) for eddy viscosity models. The analysis was extended to Reynolds stress models by

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doi:10.1016/j.ijheatfluidflow.2006.03.022

Cazalbou and Chassaing (2002). They showed that, near the interface, the problem should reduce to a convection/diffusion equilibrium and that source terms in the turbulence transport equation should be negligible.

The problem is investigated in a reference frame linked to the interface. The ordinate y is along the normal to the average interface and the relevant length is $\lambda = y_e - y$, i.e., the distance to the turbulent/non-turbulent interface. A defect form for the velocity profile is considered as $W = u_e - u$, u_e being the velocity in the non-turbulent region. In the turbulent region, the solution of the simplified transport equations for mean defect velocity W, turbulent stresses $\overline{u_i'u_j'}$, turbulent kinetic energy k and, e.g., turbulent dissipation rate ε is sought for as power laws as:

$$W = A_u\lambda^{\alpha_U} \quad \overline{u_i'u_j'} = A_{uu}\lambda^{\alpha_k} \quad k = A_k\lambda^{\alpha_k} \quad \varepsilon = A_\varepsilon\lambda^{\alpha_\varepsilon}.$$

The A are undetermined scaling factors and the exponents α should be positive for the turbulence to propagate towards the non-turbulent region and larger than unity to achieve a smooth matching with the external flow.

These power-law behaviours as well as the constraints on the coefficients were validated by numerical computations. Moreover, for Reynolds stress models, Flachard (2000) and Cazalbou and Chassaing (2002) pointed out the importance of the diffusion model in the behaviour near the interface. For example, Hanjalić and Launder (1972) diffusion model induces a strong and unrealistic turbulence anisotropy near the interface while a good behaviour is obtained with Daly and Harlow (1970) model.

3. Present analysis

3.1. Turbulence models

Problems encountered at ONERA with a model fulfilling the above constraints led to the conclusion that the standard analysis is too simple and has to be revisited. The present analysis holds as well for two-equation models, whatever the constitutive relation (eddy viscosity, non-linear eddy viscosity or explicit algebraic Reynolds stress model) as for differential Reynolds stress models. For that, generic models, following Catris and Aupoix (2000), are introduced.

Two-equation models solve two transport equations, generally for the turbulent kinetic energy k and an arbitrary length scale determining variable $\Phi = k^m\varepsilon^n$. Following Catris and Aupoix (2000), a generic model reads:

$$\frac{Dk}{Dt} = P_k - \varepsilon + \frac{\partial}{\partial x_k}\left(D_{kk}\nu_t\frac{\partial k}{\partial x_k}\right), \tag{1}$$

$$\begin{aligned}\frac{\mathrm{d}\Phi}{Dt} &= (C_{\Phi 1}P_k - C_{\Phi 2}\varepsilon)\frac{\Phi}{k} + \frac{\partial}{\partial x_k}\left(D_{\Phi\Phi}\nu_t\frac{\partial \Phi}{\partial x_k}\right)\\ &\quad + \frac{\partial}{\partial x_k}\left(D_{\Phi k}\frac{\nu_t\Phi}{k}\frac{\partial k}{\partial x_k}\right) + C_{\Phi\Phi}\frac{\nu_t}{\Phi}\frac{\partial \Phi}{\partial x_k}\frac{\partial \Phi}{\partial x_k}\\ &\quad + C_{\Phi k}\frac{\nu_t}{k}\frac{\partial k}{\partial x_k}\frac{\partial \Phi}{\partial x_k} + C^{\Phi}_{kk}\frac{\nu_t\Phi}{k^2}\frac{\partial k}{\partial x_k}\frac{\partial k}{\partial x_k}.\end{aligned} \tag{2}$$

This is a generic form as the transport equation for any combination of k and Φ, deduced from the above equations, has the same form as Eq. (2), its constants being linked to those of Eqs. (1) and (2). Compared to the generic form proposed by Catris and Aupoix (2000), a diffusion term was dropped in the turbulent kinetic energy transport equation for the sake of simplicity. These transport equations have to be coupled with a constitutive relation, of the eddy viscosity form, either linear or non-linear.

Similarly, for differential Reynolds stress models, the generic transport equations read:

$$\frac{D\overline{u_i'u_j'}}{Dt} = P_{ij} + \Pi_{ij} - \varepsilon_{ij} + \frac{\partial}{\partial x_k}\left(D_{kk}\frac{k}{\varepsilon}\overline{u_k'u_l'}\frac{\partial\overline{u_i'u_j'}}{\partial x_l}\right), \tag{3}$$

$$\begin{aligned}\frac{D\Phi}{Dt} &= (C_{\Phi 1}P_k - C_{\Phi 2}\varepsilon)\frac{\Phi}{k} + \frac{\partial}{\partial x_k}\left(D_{\Phi\Phi}\frac{k}{\varepsilon}\overline{u_k'u_l'}\frac{\partial \Phi}{\partial x_l}\right)\\ &\quad + \frac{\partial}{\partial x_k}\left(D_{\Phi k}\frac{\Phi}{\varepsilon}\overline{u_k'u_l'}\frac{\partial k}{\partial x_l}\right) + C_{\Phi\Phi}\frac{k}{\varepsilon\Phi}\frac{\partial \Phi}{\partial x_k}\overline{u_k'u_l'}\frac{\partial \Phi}{\partial x_l}\\ &\quad + C_{\Phi k}\frac{1}{\varepsilon}\frac{\partial k}{\partial x_k}\overline{u_k'u_l'}\frac{\partial \Phi}{\partial x_l} + C^{\Phi}_{kk}\frac{\Phi}{k\varepsilon}\frac{\partial k}{\partial x_k}\overline{u_k'u_l'}\frac{\partial k}{\partial x_l},\end{aligned} \tag{4}$$

where P_{ij}, Π_{ij} and ε_{ij}, respectively stand for the Reynolds stress production, redistribution and destruction terms the form of which is not of concern here. Following Cazalbou and Chassaing (2002) conclusions, the analysis is restricted to Daly and Harlow (1970) diffusion model.

3.2. Equations in the vicinity of the turbulent region edge

A complete derivation of the equations can be found in Ferrey (2004). Either a time evolving flow, as in Cazalbou et al. (1994) or a spatially evolving flow, as here, can be considered. For the analysis, it is more convenient to use, as length scale determining variable, the eddy viscosity ν_t or $\frac{\overline{v'^2}k}{\varepsilon}$ for Reynolds stress models. Considering a two-equation model, a two-dimensional steady flow, neglecting viscosity and introducing $g = -\frac{D_{kk}\nu_t}{V_0}$, where V_0 is the velocity component normal to the interface, the system of equations for the momentum and transport equations reduces to:

$$D_{kk}\frac{\mathrm{d}W}{\mathrm{d}\lambda} = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(g\frac{\mathrm{d}W}{\mathrm{d}\lambda}\right) \tag{5}$$

$$\frac{\mathrm{d}k}{\mathrm{d}\lambda} = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(g\frac{\mathrm{d}k}{\mathrm{d}\lambda}\right) \tag{6}$$

$$\begin{aligned}D_{kk}\frac{\mathrm{d}g}{\mathrm{d}\lambda} &= \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(D_{\nu_t\nu_t}g\frac{\mathrm{d}g}{\mathrm{d}\lambda} + D_{\nu_t k}\frac{g^2}{k}\frac{\mathrm{d}k}{\mathrm{d}\lambda}\right) + C_{\nu_t\nu_t}\left(\frac{\mathrm{d}g}{\mathrm{d}\lambda}\right)^2\\ &\quad + C_{\nu_t k}\frac{g}{k}\frac{\mathrm{d}g}{\mathrm{d}\lambda}\frac{\mathrm{d}k}{\mathrm{d}\lambda} + C^{\nu_t}_{kk}\left(\frac{g}{k}\frac{\mathrm{d}k}{\mathrm{d}\lambda}\right)^2.\end{aligned} \tag{7}$$

In the above equations, the problem was assumed to reduce to an advection/diffusion equilibrium, so that source terms were dropped in the turbulence transport equation. This has however to be checked and leads to other constraints for the model (see, e.g., Cazalbou et al. (1994), Hellsten and Bézard (2005)), not to be discussed here.

The boundary conditions at the interface are:

$$\lim_{\lambda\to 0} W = 0 \quad \lim_{\lambda\to 0} k = 0 \quad \lim_{\lambda\to 0} g = 0. \tag{8}$$

Integrating Eq. (6) with the above boundary conditions yields:

$$k = g\frac{\mathrm{d}k}{\mathrm{d}\lambda} \Rightarrow g = -\frac{D_{kk}}{V_0}\nu_t = \frac{k}{\frac{\mathrm{d}k}{\mathrm{d}\lambda}}. \tag{9}$$

Introducing the above relation into Eq. (7) leads to the following relation:

$$ag\frac{\mathrm{d}^2 g}{\mathrm{d}\lambda^2} + b\left(\frac{\mathrm{d}g}{\mathrm{d}\lambda}\right)^2 + c\frac{\mathrm{d}g}{\mathrm{d}\lambda} + d = 0 \tag{10}$$

where the coefficients a, b, c and d are linked to the diffusion coefficients C and D as:

$$a = -D_{\nu_t\nu_t}, \quad b = -(D_{\nu_t\nu_t} + C_{\nu_t\nu_t}),$$
$$c = -(C_{\nu_t k} + D_{\nu_t k} - D_{kk}), \quad d = -C^{\nu_t}_{kk}. \tag{11}$$

As model coefficients are interrelated when the length scale determining variable is changed, these coefficients can be expressed, referring to a $k-\varepsilon$ model form, as:

$$a = -D_{\varepsilon\varepsilon}, \quad b = D_{\varepsilon\varepsilon} + C_{\varepsilon\varepsilon},$$
$$c = -4(D_{\varepsilon\varepsilon} + C_{\varepsilon\varepsilon}) + D_{kk} - D_{\varepsilon k} - C_{\varepsilon k},$$
$$d = 4(D_{\varepsilon\varepsilon} + C_{\varepsilon\varepsilon}) + 2(D_{\varepsilon k} + C_{\varepsilon k} - D_{kk}) + C^{\varepsilon}_{kk}. \tag{12}$$

The same analysis holds for a Reynolds stress model, using Daly and Harlow diffusion. This is due to the fact, pointed out by Cazalbou and Chassaing (2002) that, using this diffusion model, the anisotropy levels are constant in the turbulent region below the turbulent/non-turbulent interface. Therefore, the same expressions for the a, b, c and d coefficients are retrieved.

The analysis is also similar for non-linear eddy viscosity models or explicit algebraic Reynolds stress models since they predict an isotropic state near the interface where the velocity gradient $\frac{\partial u}{\partial y}$ tends towards zero and reduce to a modified eddy viscosity model for the turbulent shear stress $-\overline{u'v'}$.

It must be pointed out that, to prevent counter-gradient diffusion, a has to be negative. Moreover, writing the standard model balance in the logarithmic region, it is easily checked that b is positive.

3.3. Solutions near the turbulent region edge

Eq. (10) has two obvious solutions such that $\frac{\mathrm{d}^2 g}{\mathrm{d}\lambda^2} = 0$, which correspond to a linear profile for the eddy viscosity. From relation (9), these solutions correspond to power law solutions for the turbulent kinetic energy profile. They read:

$$g = \frac{\lambda}{\alpha_k^+} \Rightarrow k = K_0^+\lambda^{\alpha_k^+} \quad g = \frac{\lambda}{\alpha_k^-} \Rightarrow k = K_0^-\lambda^{\alpha_k^-},$$
$$\frac{1}{\alpha_k^+} = -\frac{c+\sqrt{\Delta}}{2b} \quad \frac{1}{\alpha_k^-} = -\frac{c-\sqrt{\Delta}}{2b} \quad \Delta = c^2 - 4bd. \tag{13}$$

The power law solution found by Cazalbou et al. (1994) is retrieved together with a second power law solution which was already pointed out by Catris (1999) and Catris and Aupoix (2000), to be discussed later.

As the transformed eddy viscosity g is linear, it must be pointed out that the length scale determining variable and turbulent kinetic energy evolutions are linked, as pointed out by Daris (2002) and Aupoix et al. (2003), as:

$$2\alpha_k^\pm - \alpha_\varepsilon^\pm = 1. \tag{14}$$

Therefore, since $\Phi = k^m\varepsilon^n$, its behaviour is given by:

$$\alpha_\Phi^\pm = (m+2n)\alpha_k^\pm - n. \tag{15}$$

The case $b = 0$ has not to be considered as the model would not satisfy the logarithmic region. The case $\Delta = 0$ also has to be rejected, as shown by Ferrey (2004), as it leads to abrupt matching at the interface.

The new and important point is that Eq. (10) has a third solution, which can only be expressed in parametric form, the parameter being $\mu = \frac{\mathrm{d}g}{\mathrm{d}\lambda}$, as:

$$g(\mu) = G_0\left|\mu - \frac{1}{\alpha_k^+}\right|^{\beta^+}\left|\mu - \frac{1}{\alpha_k^-}\right|^{\beta^-}$$
$$\beta^+ = \frac{a}{\sqrt{\Delta}\alpha_k^+} \quad \beta^- = -\frac{a}{\sqrt{\Delta}\alpha_k^-}, \tag{16}$$

$$\lambda(\mu) = c_1\left|\mu - \frac{1}{\alpha_k^-}\right|^{\beta^-} F\left(\mu - \frac{1}{\alpha_k^-}\right) \quad \text{with}$$
$$F(x) = \mathrm{Hypergeom}\left(\beta^-, 1-\beta^+, 1+\beta^-; \frac{\alpha_k^+\alpha_k^- x}{\alpha_k^+ - \alpha_k^-}\right), \tag{17}$$

where Gauss' hypergeometric function is defined as:

$$\mathrm{Hypergeom}(a,b,c;z) = \sum_{k=0}^{+\infty}\frac{\Gamma(a+k)}{\Gamma(a)}\frac{\Gamma(b+k)}{\Gamma(b)} \times \frac{\Gamma(c)}{\Gamma(c+k)}\frac{z^k}{k!} \tag{18}$$

and involves Euler's Γ function

$$\Gamma(z) = \int_0^\infty t^{z-1}\mathrm{e}^{-t}\mathrm{d}t = (z-1)! \tag{19}$$

which extends the factorial to real (and complex) number arguments.

G_0 and c_1 are constants the values of which are given by the matching with the – non considered here – solution far from the interface.

Similarly, the turbulent kinetic energy profile can be expressed as:

$$k(\mu) = K_0\left|\frac{\mu - \frac{1}{\alpha_k^-}}{\mu - \frac{1}{\alpha_k^+}}\right|^{-\frac{a}{\sqrt{\Delta}}}. \tag{20}$$

The velocity profile can be linked to the turbulent kinetic energy profile. For eddy viscosity models, Eqs. (5) and (6) can be integrated, using boundary conditions (8), as:

$$D_{kk}W = g\frac{\mathrm{d}W}{\mathrm{d}\lambda} \quad k = g\frac{\mathrm{d}k}{\mathrm{d}\lambda}, \tag{21}$$

so that, eliminating g,

$$W = k^{D_{kk}}. \tag{22}$$

For Reynolds stress models, the momentum equation reduces to:

$$V_0 \frac{\mathrm{d}W}{\mathrm{d}\lambda} = -\frac{\mathrm{d}\overline{u'v'}}{\mathrm{d}\lambda}, \tag{23}$$

so that, as the anisotropy levels tend towards constants near the interface:

$$W \propto \overline{u'v'} \propto k. \tag{24}$$

3.4. Behavior of the parametric solution

This parametric solution is the only relevant solution as a small perturbation of the linear solutions of Eq. (10) will lead to $\frac{\mathrm{d}^2g}{\mathrm{d}\lambda^2} \neq 0$ and thus to the parametric solution.

The boundary conditions at the interface (8) and the above solutions for k (20) or λ (17) shows that the interface corresponds to $\mu = \frac{1}{\alpha_k^-}$. As, near the interface, V_0 is negative and therefore g is positive and tends towards zero at the interface, $\mu = \frac{\mathrm{d}g}{\mathrm{d}\lambda}$ must be positive so that α_k^- must be positive.

Two cases must be considered, whether α_k^+ is positive or negative, its sign being that of d. If α_k^+ is positive, it is obvious from its definition (13) that $\alpha_k^+ > \alpha_k^-$. Two behaviours are possible, according to the sign of $\frac{\mathrm{d}\mu}{\mathrm{d}\lambda} = \frac{\mathrm{d}^2g}{\mathrm{d}\lambda^2}$. If $\frac{\mathrm{d}\mu}{\mathrm{d}\lambda}$ is negative, μ decreases from $\frac{1}{\alpha_k^-}$. When μ tends towards $\frac{1}{\alpha_k^+}$, as a is positive and thus β^+ is negative, according to (16), the transformed eddy viscosity tends towards infinity. As $\mu = \frac{\mathrm{d}g}{\mathrm{d}\lambda}$, remains finite, λ also tends towards infinity. Moreover, it can be shown that the g evolution asymptotes the two power law solutions (13) for $\lambda \to 0$ and $\lambda \to \infty$. This is the situation depicted in the left part of Fig. 1. If $\frac{\mathrm{d}\mu}{\mathrm{d}\lambda}$ is positive, μ and λ increase and g rapidly blows out. The g evolution then only has one asymptote for $\lambda \to 0$.

Similar behaviours are observed if α_k^+ is negative. If μ decreases, the g profile asymptotes both power law solutions, so that g rapidly becomes negative while it blows out if μ increases, as shown in Fig. 2.

Unfortunately, no way to determine the sign of $\frac{\mathrm{d}\mu}{\mathrm{d}\lambda}$ was found.

It must be pointed out that the two power law solutions, which are not relevant solutions, are in some way included in the parametric solution as they provide asymptotic behaviours.

3.5. Numerical solutions

The occurrence and hence the relevance of this parametric solution was checked using a code solving the self-similarity equations for various simple flows such as the outer region of the boundary layer, the wake, the mixing layer and the plane or round jets (Bézard, 2000). As self-similarity reduces the equation set to a one-dimensional problem, which is solved using a time marching technique, grid convergence is easily achieved. An example of result is provided in Fig. 3 where the "eddy viscosity" $\frac{\overline{v'^2}k}{\varepsilon}$ is plotted for the outer region of the boundary layer, using a Reynolds stress model for which both α_k are positive. The lower figure shows the eddy viscosity profile in self similar coordinates ($\frac{\nu_t}{u_e\Delta}$ versus $\eta = \frac{y}{\Delta}$, where Δ is here the Clauser's thickness) while the upper figure shows an enlargement of the solution near the interface. The two linear solutions for the eddy viscosity are also plotted. It can be checked that the parametric solution is retrieved and that it asymptotes both linear laws, as in Fig. 1(left). As the α_k^- solution is reached only in the very vicinity of the interface, it explains why previous works concluded that the α_k^+ solution was obtained.

As in Cazalbou et al. (1994) analysis, fluid viscosity was neglected. When it is accounted for, or when a small level of turbulence is present in the irrotational flow, the α_k^- solution is difficult to observe as it is superseded by the viscous effects or the free-stream condition. Moreover, the α_k^- solution generally extends over a very restricted (if not null) number of cells with usual grids.

For models for which the cross diffusion coefficient C_{kk}^{Φ} is null, which involves most of the classical models, Cazalbou's solution corresponds to the α_k^+ solution. The other power law solution, which gives the tangent for $\lambda = 0$,

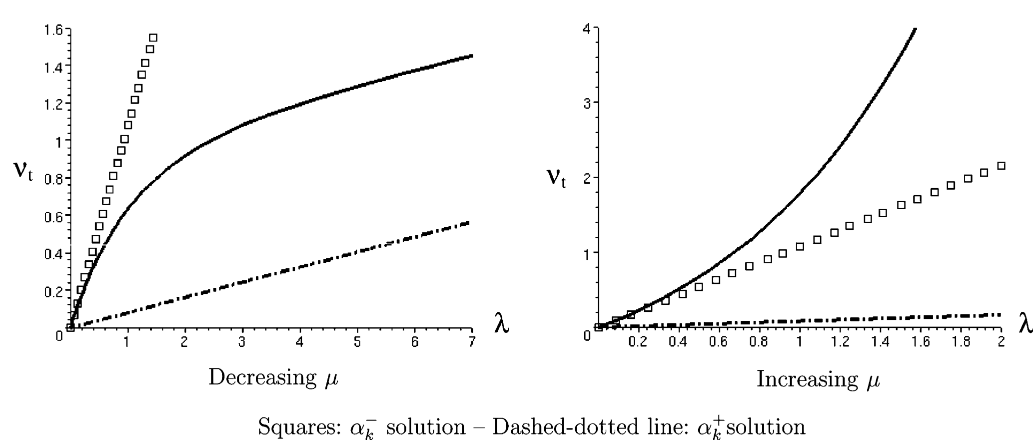
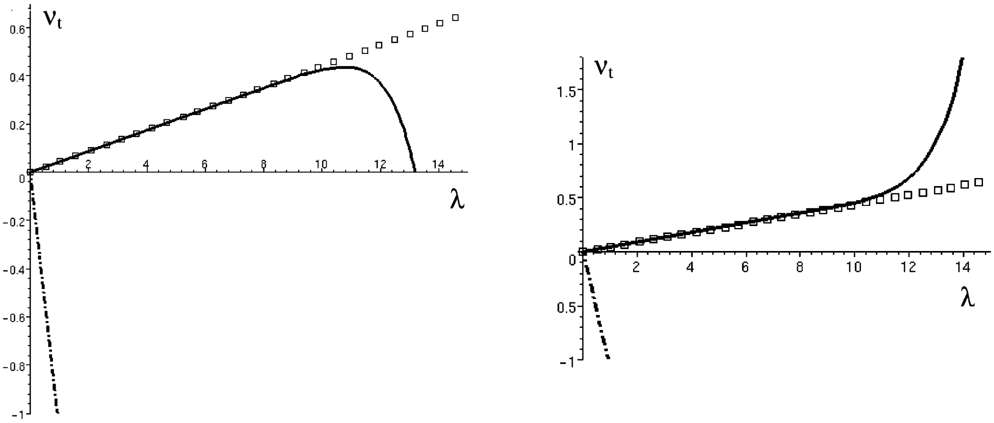


Fig. 1. Solutions for the eddy viscosity profile near the interface when both exponents are positive.



Decreasing μ Increasing μ

Squares: α_k^- solution – Dashed-dotted line: α_k^+solution

Fig. 2. Solutions for the eddy viscosity profile near the interface when only one exponent is positive.

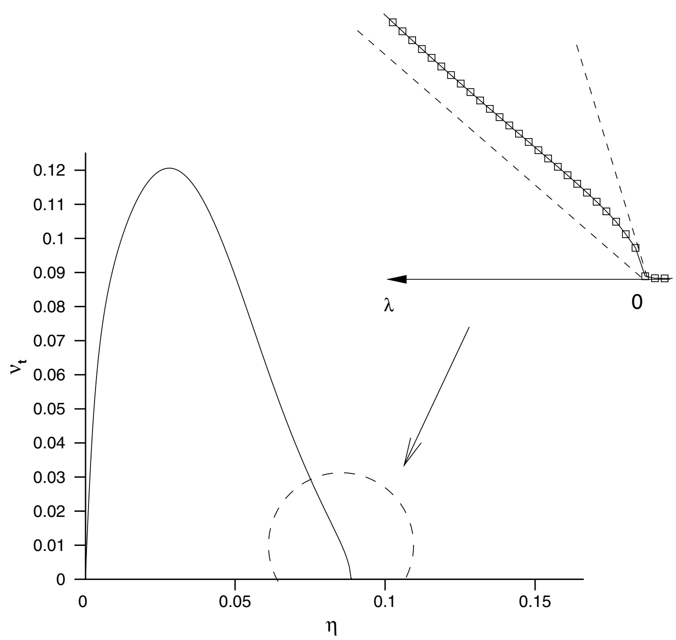


Fig. 3. Solution of the "eddy viscosity" profile in the vicinity of the interface when the exponents α are positive ($\eta \propto \frac{y}{\delta}$).

corresponds to $\alpha_\Phi^- = 0$ and $\alpha_k^- = \frac{n}{2n+m}$ (e.g., $\alpha_k^- = \frac{1}{2}$ for standard $k-\varepsilon$ models, $\alpha_k^- = 1$ for $k-\omega$ models). Therefore, the length scale determining variable $\Phi = k^m\varepsilon^n$ smoothly tends towards zero, as the power law behaviour is only retrieved at the interface. It must be pointed out that $\alpha_\Phi^- = 0$ does not lead to specific problems. This only gives the limiting behaviour when $\lambda \to 0$, i.e., at the interface: Φ thus asymptotes a constant value, which is null, i.e., it goes to zero very smoothly.

4. Consequences for turbulence models

The first important point is that not only α_k^+ has to be considered, as previously done by Cazalbou et al. (1994) but also α_k^- and that α_k^- also must be positive. This explains why Cazalbou et al. (1994) were unable to apply their analysis to the Ng and Spalding model: among the models they considered, it was the only one for which α_k^- is negative.

Cazalbou et al. (1994) recommended that α_k^+ be positive to have the correct information propagation. Among the four possible behaviours depicted in Figs. 1 and 2, it seems that the case where α_k^+ is positive and μ is decreasing, where the g evolution asymptotes the α_k^+ power law, is the only acceptable case. In all other cases, g blows out and a matching with the g profile in the turbulent region, where production and destruction terms are no longer negligible, seems more problematic. Indeed, this is this behaviour that was retrieved in all the numerical simulations.

Therefore, it seems that the constraint should be that both α_k must be positive, i.e.,

$$\frac{1}{\alpha_k^-} > 0 \quad \frac{1}{\alpha_k^+} > 0, \tag{25}$$

where inverses are used to discard infinite values for the α_k, as suggested by Cazalbou (private communication).

Numerical checks tend to support the above conclusion. As an example, "eddy viscosity" ($\frac{\overline{v'^2}k}{\varepsilon}$) profiles are plotted for the outer region of a boundary layer, using Reynolds stress models. Different solutions are obtained, always imposing a very small turbulence level outside of the boundary layer but varying the "eddy viscosity" level. In Fig. 4, both α_k are positive and the solution in the boundary layer is insensitive to the imposed "eddy viscosity" level outside. Turbulence propagates from the turbulent region towards the non-turbulent region so that the eddy viscosity first falls to a very small value near the interface before rising again to the value imposed in the external flow. In Fig. 5, α_k^+ is negative and the solution is deeply affected by the external "eddy viscosity" level. Turbulence propagates from the external flow into the turbulent region, which is not wanted. It should be pointed out that the above analysis can no longer be strictly applied since the "eddy viscosity" thus no longer tends towards zero at the boundary layer edge.

Moreover, for each transported quantity (turbulent kinetic energy, Reynolds stress tensor component, length scale determining variable) the exponent α should be such that source terms are negligible compared to the advection and diffusion terms. This leads to very different constraints according to the constitutive relation, as pointed out by Hellsten and Bézard (2005). Provided the parametric solution is bounded by the two power law solutions, the analysis is thus similar to the one proposed by Cazalbou et al. (1994).

A last point to be mentioned is that the generic character is in some sense lost near the interface. In other words, the choice of the length scale determining variable must be done carefully since, rewriting a model to change the length scale determining variable may affect its behaviour. Indeed, from Eq. (15), the exponents of the turbulent kinetic energy and of the length scale determining variable

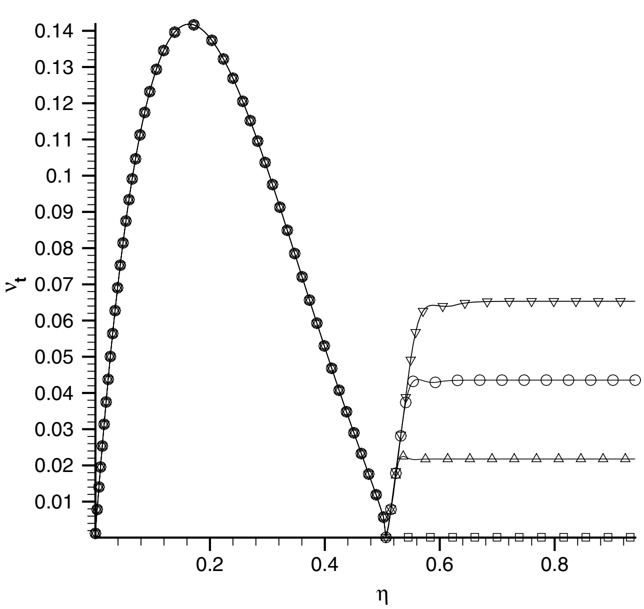


Fig. 4. Sensitivity to free-stream values of the eddy viscosity of a model such that all α are positive.

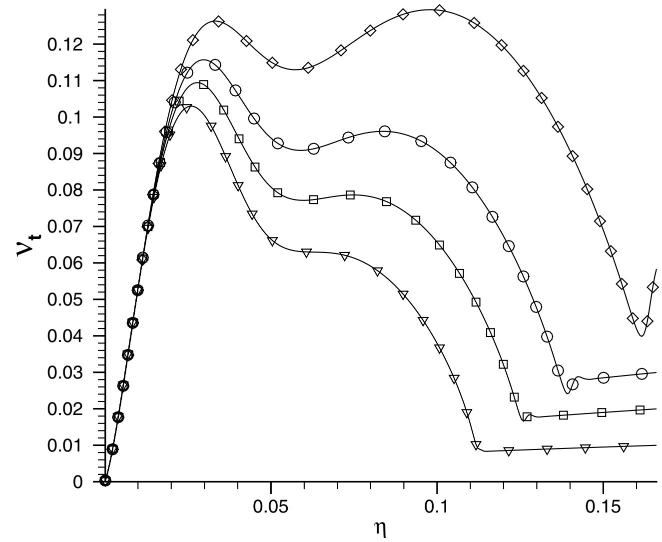


Fig. 5. Sensitivity to free-stream values of the eddy viscosity of a model such that all α are not positive.

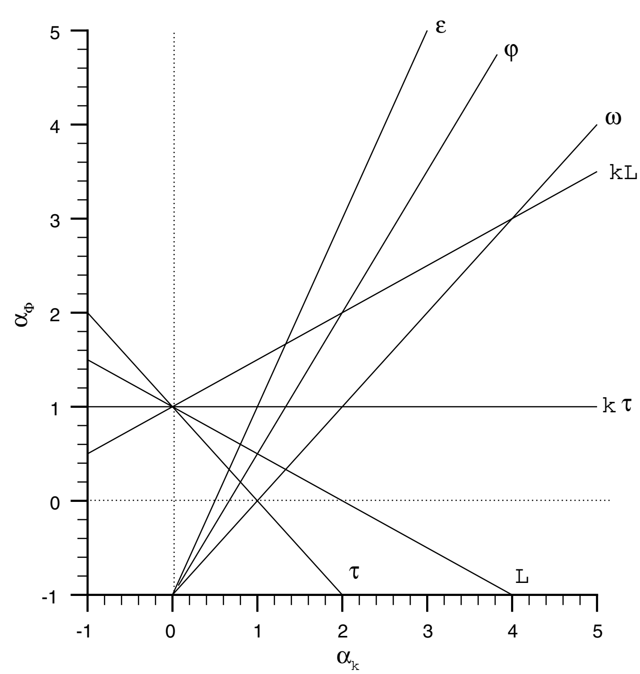


Fig. 6. Behaviours of the exponents α for several length-scale determining variables.

are interrelated. This link between the exponents α is shown in Fig. 6 for some popular length scale determining variables ($\omega \propto \frac{\varepsilon}{k}, \varphi \propto \frac{\varepsilon}{\sqrt{k}}, \tau \propto \frac{k}{\varepsilon}, L \propto \frac{k^{3/2}}{\varepsilon}$). If only a positive value of the exponents α_k and α_Φ for the length scale determining variable is looked for, which ensures a correct information propagation, any length scale determining variable can be used, with caution. If at least α_k^+ and α_Φ^+ are imposed to be higher than unity, to ensure a smooth matching with the small external level, $\tau \propto \frac{k}{\varepsilon}$ or $l \propto \frac{k^{3/2}}{\varepsilon}$ are forbidden. This last consideration thus leads to the final set of constraints:

$$\frac{1}{\alpha_k^-} > 0 \quad \alpha_k^+ \geqslant 1 \quad (m+2n)\alpha_k^+ \geqslant 1+n, \tag{26}$$

which is recommended.

5. Conclusions

The above analysis shows that the behaviour of a turbulence model near a turbulent/non-turbulent interface is more complex than previously considered. The real solution is not given by power laws but is a parametric solution, which generally seems to asymptote the power laws. This explains why the numerical solutions were confused with the power law solutions.

The above analysis holds as well for two-equation models, whatever the constitutive relation, as for Reynolds stress models. It leads to a more complex constraint, imposing that both α_k coefficients must be positive.

The analysis still requires to be extended, on the one hand to be able to determine which solution, i.e., increasing or decreasing μ is obtained and why and, on the other hand, to account for viscosity and small external turbulence levels.

Acknowledgements

The authors wish to thank DGA (French Ministry of Defence) which granted P. Ferrey's thesis. Part of this work took place within the FLOMANIA project (Flow Physics Modelling – An Integrated Approach) which is a collaboration between Alenia, Ansys-CFX, Bombardier, Dassault, EADS-CASA, EADS-Military Aircraft, EDF, NUMECA, DLR, FOI, IMFT, ONERA, Chalmers University, Imperial College, TU Berlin, UMIST and St. Petersburg State University. This project is funded by the European Union and administrated by the CEC, Research Directorate-general, Growth Programme, under Contract No. G4RD-CT2001-00613.

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